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Soliton dynamics in two coupled ferromagnetic chains

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Abstract. The soliton dynamics in a system of two ferromagnetic chains coupled through the interaction between opposite spins is studied. The on-site anisotropy in the chains is also taken into account. The conditions for the existence of different soliton solutions (bright or dark) in the chains are obtained. The system is reduced to a coupled set of discrete equations with complicated coupling interactions which are linear and nonlinear. We investigate the propagation of a soliton excitation launched in one of the chains with a given velocity. The condition for a perfect switch of the soliton is analysed. Numerical simulations have been performed for a variety of the magnetic parameters and the soliton characteristics.

1. Introduction

The study of solitary waves in solids has been subject of considerable attention for decades. A large amount of theoretical and experimental research has been dedicated to one-dimensional magnetic systems [1]. Models corresponding to real quasi-one-dimensional magnets are broadly investigated as first they were considered as somewhat simpler than the really interesting three-dimensional systems but then turned out to be interesting in their own right. There was the availability of real magnetic compounds which because of extremely small coupling between neighboring chains, could be considered as reasonable realizations of magnetically one-dimensional materials. Soliton solutions for classical sine-Gordon chains [2] and Heisenberg chains with various anisotropies [3-6] were obtained and analyzed. In recent years there is a renewed interest to the topic due to their application. Spin-wave dark solitons were predicted and experimentally generated [7,8]. Solitons in magnetic thin films [9] and in ferromagnets with biquadratic exchange [10] were investigated.

Bright and dark solitons as exact solutions of the nonlinear Schrödinger (NLS) equation have been studied intensively [3,4,11,12]. Another interesting topic of research with practical importance is the propagation of solitons in coupled parallel chains [13-20].

In the present paper we study the dynamics of bright solitons in two anisotropic ferromagnetic chains coupled through a complicated interaction.

2. Hamiltonian of the system

We consider two ferromagnetic Heisenberg chains of N spins with magnitude S described in the nearest-neighbors approximation by the following Hamiltonian:

$$\hat{H} = -J \sum_{n=1}^N (\hat{\vec{S}}_n \cdot \hat{\vec{S}}_{n+1} + \hat{\vec{\sigma}}_n \cdot \hat{\vec{\sigma}}_{n+1}) - A \sum_{n=1}^N [(\hat{S}_n^z)^2 + (\hat{\sigma}_n^z)^2] - \mu H_0 \sum_{n=1}^N (\hat{S}_n^z + \hat{\sigma}_n^z) + d \sum_{n=1}^N \hat{\vec{S}}_n \cdot \hat{\vec{\sigma}}_n, \quad (1)$$



where $J > 0$ is the exchange integral and A is the on-site anisotropy constant which can be positive (easy axis) or negative (easy plane). H_0 is the external magnetic field applied along the z -axis, so that in the ground state of the system all spins are aligned in the z -direction, μ is the magnetic moment per spin, d characterizes the coupling interaction between the two chains.

We use for the scalar products in (1) the rule

$$\hat{\vec{a}} \cdot \hat{\vec{b}} = \frac{\hat{a}^+ \hat{b}^- + \hat{a}^- \hat{b}^+}{2} + \hat{a}^z \hat{b}^z, \quad (2)$$

$$[\hat{S}_i^\pm, \hat{S}_j^z] = \mp \hat{S}_i^\pm \delta_{ij}, \quad [\hat{S}_i^+, \hat{S}_j^-] = 2\hat{S}_i^z \delta_{ij}, \quad [\hat{\sigma}_i^\pm, \hat{\sigma}_j^z] = \mp \hat{\sigma}_i^\pm \delta_{ij}, \quad [\hat{\sigma}_i^+, \hat{\sigma}_j^-] = 2\hat{\sigma}_i^z \delta_{ij}. \quad (3)$$

All the other commutators are zero.

The equations of motion for $\hat{S}_i^\pm$ and $\hat{\sigma}_i^\pm$

$$i\hbar \dot{\hat{a}} = [\hat{a}, \hat{H}] \quad (4)$$

yield in our case:

$$\begin{aligned} \pm i\hbar \dot{\hat{S}}_i^\pm &= \mu H_0 \hat{S}_i^\pm - J[\hat{S}_i^z(\hat{S}_{i-1}^\pm + \hat{S}_{i+1}^\pm) - \hat{S}_i^\pm(\hat{S}_{i-1}^z + \hat{S}_{i+1}^z)] + A(\hat{S}_i^z \hat{S}_i^\pm + \hat{S}_i^\pm \hat{S}_i^z) + d(\hat{S}_i^z \hat{\sigma}_i^\pm - \hat{S}_i^\pm \hat{\sigma}_i^z), \\ \pm i\hbar \dot{\hat{\sigma}}_i^\pm &= \mu H_0 \hat{\sigma}_i^\pm - J[\hat{\sigma}_i^z(\hat{\sigma}_{i-1}^\pm + \hat{\sigma}_{i+1}^\pm) - \hat{\sigma}_i^\pm(\hat{\sigma}_{i-1}^z + \hat{\sigma}_{i+1}^z)] + A(\hat{\sigma}_i^z \hat{\sigma}_i^\pm + \hat{\sigma}_i^\pm \hat{\sigma}_i^z) - d(\hat{S}_i^z \hat{\sigma}_i^\pm - \hat{S}_i^\pm \hat{\sigma}_i^z). \end{aligned} \quad (5)$$

In the quasiclassical approximation, where the components of the spin operators are complex amplitudes, $\alpha_n = \hat{S}_n^+/S$, $\alpha_n^* = \hat{S}_n^-/S$, $\hat{S}_n^z/S = \sqrt{1 - |\alpha_n|^2}$, $\beta_n = \hat{\sigma}_n^+/S$, $\beta_n^* = \hat{\sigma}_n^-/S$ and $\hat{\sigma}_n^z/S = \sqrt{1 - |\beta_n|^2}$, we have

$$\begin{aligned} i\hbar \frac{\partial \alpha_n}{\partial t} &= \mu H_0 \alpha_n - JS \left[(\alpha_{n+1} + \alpha_{n-1}) \sqrt{1 - |\alpha_n|^2} - \alpha_n \left(\sqrt{1 - |\alpha_{n+1}|^2} + \sqrt{1 - |\alpha_{n-1}|^2} \right) \right] \\ &+ 2AS\alpha_n \sqrt{1 - |\alpha_n|^2} + dS \left(\beta_n \sqrt{1 - |\alpha_n|^2} - \alpha_n \sqrt{1 - |\beta_n|^2} \right), \\ i\hbar \frac{\partial \beta_n}{\partial t} &= \mu H_0 \beta_n - JS \left[(\beta_{n+1} + \beta_{n-1}) \sqrt{1 - |\beta_n|^2} - \beta_n \left(\sqrt{1 - |\beta_{n+1}|^2} + \sqrt{1 - |\beta_{n-1}|^2} \right) \right] \\ &+ 2AS\beta_n \sqrt{1 - |\beta_n|^2} + dS \left(\alpha_n \sqrt{1 - |\beta_n|^2} - \beta_n \sqrt{1 - |\alpha_n|^2} \right). \end{aligned} \quad (6)$$

The set of differential equations (6) describes our system.

3. Soliton solutions

We shall look for solutions in the form of amplitude-modulated waves

$$\alpha_n(t) = \varphi_n(t) e^{i(kn - \omega t)}, \quad \beta_n(t) = \psi_n(t) e^{i(kn - \omega t)}, \quad (7)$$

where k and ω are the wave number and the frequency of the carrier waves (the lattice constant equals unity) and the envelopes $\varphi_n(t)$, $\psi_n(t)$ are slowly varying functions of the position and time. In the continuum limit, when the soliton width is much larger than the lattice spacing ($L \gg 1$), we use $\varphi \sim O(1)$, $\frac{\partial \varphi}{\partial x} \sim O(2)$ and $\frac{\partial^2 \varphi}{\partial x^2} \sim O(3)$ and remain only $O(3)$ terms. Then, equations (6) transform into the following coupled modified NLS equations for the envelopes:

$$\begin{aligned} i \left(\hbar \frac{\partial \varphi}{\partial t} + 2J \sin k \frac{\partial \varphi}{\partial x} \right) &= (\varepsilon - \hbar \omega S^{-1}) \varphi - J \cos k \frac{\partial^2 \varphi}{\partial x^2} + g|\varphi|^2 \varphi + \frac{d}{2} (2\psi - |\varphi|^2 \psi + |\psi|^2 \varphi), \\ i \left(\hbar \frac{\partial \psi}{\partial t} + 2J \sin k \frac{\partial \psi}{\partial x} \right) &= (\varepsilon - \hbar \omega S^{-1}) \psi - J \cos k \frac{\partial^2 \psi}{\partial x^2} + g|\psi|^2 \psi + \frac{d}{2} (2\varphi - |\psi|^2 \varphi + |\varphi|^2 \psi), \end{aligned} \quad (8)$$

where

$$\varepsilon = \mu H_0 S^{-1} - 2g - d, \quad g = J(\cos k - 1 - A/J). \quad (9)$$

For $d = 0$ the uncoupled equations (8) possess soliton solutions of different types depending on the sign of the expression $gJ \cos k$. For negative values bright solitons exist while for positive values dark solitons are possible. It is interesting to point out that for the isotropic case ($A = 0$) equations (8) have bright-soliton solutions for $0 < k < \pi/2$ and dark-soliton solutions for $\pi/2 < k \leq \pi$. For the anisotropic case ($A \neq 0$) the situation is more complicated (figure 1).

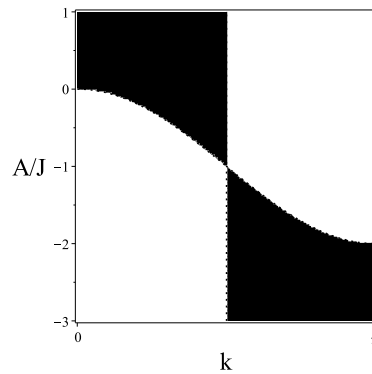


Figure 1. Regions in the Brillouin zone for the existence of bright (shaded region) and dark (light region) solitons depending on A/J .

In what follows, we shall consider bright-soliton solutions which appear for

$$\cos k(\cos k - 1 - A/J) < 0 \quad (10)$$

and $|\varphi(x)|^2 \rightarrow 0, |\psi(x)|^2 \rightarrow 0$ at $x \rightarrow \pm\infty$. Note that the condition (10) depends not only on the anisotropy constants but also on the wave number k . Insert

$$\varphi(x, t) = \varphi_0 \operatorname{sech} \frac{x - vt}{L}, \quad \psi(x, t) = \psi_0 \operatorname{sech} \frac{x - vt}{L} \quad (11)$$

in (8) one gets

$$\varphi_0^2 = \psi_0^2 = -\frac{2J \cos k}{gL^2}, \quad \hbar\omega = \varepsilon S - \frac{JS \cos k}{L^2}, \quad v = \frac{2JS}{\hbar} \sin k, \quad (12)$$

where L and v are soliton's width and velocity.

4. Numerical results

We shall investigate the evolution of a soliton which at the initial time $t = 0$ is launched in one of the chains at the position n_0

$$\alpha_n(0) = \frac{1}{L} \sqrt{\frac{2 \cos k}{A/J + 1 - \cos k}} \operatorname{sech} \frac{n - n_0}{L} e^{ikn}, \quad \beta_n(0) = 0 \quad (13)$$

solving numerically the system (6). The simulations are carried out for large enough chains compared to the soliton width L and periodic boundary conditions.

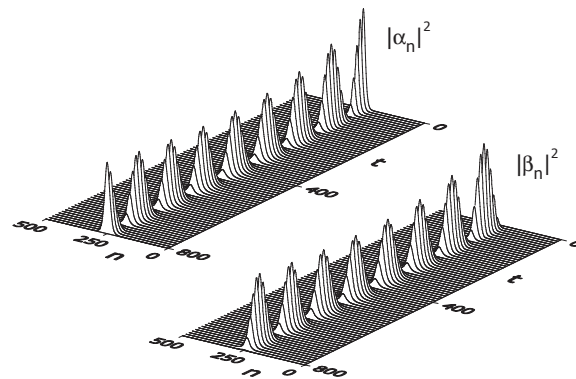


Figure 2. Perfect soliton switching for $J = S = 1$, $L = 10$, $A = 1$ and $|d| = 0.0314$. The time is in units of $\hbar/JS$.

In the linear case the excitation would be transferred from one chain to the other and back with a period $t_0 = \pi/|d|$, where d is the total linear coupling. In our complicated case a soliton can be transferred with the same period (perfect soliton switching) when the condition

$$\left| \frac{J \cos k}{2L^2 d} \right| \ll 1 \quad (14)$$

is fulfilled (figure 2). There is a critical value d_c below which the soliton will be not transferred. It depends on the wave number k i.e. the soliton velocity and when the velocity increases ($k \rightarrow \pi/2$) we have $d_c \rightarrow 0$ (figure 3).

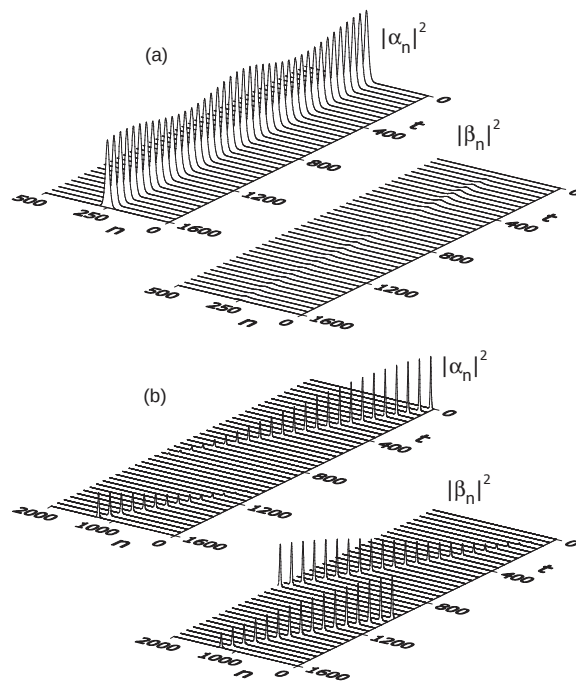


Figure 3. Soliton propagation in the two chains for $A = 1$, $d = 0.00157$ and different values of the wave number k . $k = 0$ (a) and $k = 1.5$ (b).

Let us point that our study is when the soliton amplitude is small ($\varphi_0^2 \ll 1$). First we consider only linear terms in the last bracket in (8). Further we have performed calculations to estimate the influence of the variety of nonlinear interactions in the system on the process of soliton switching. There are two types of nonlinear interchain interactions: Type I connected with the first term and type II connected with the second term proportional to d in the equations (6) or (8). Figures 4 and 5 illustrate the results for both positive and negative values of d . The elimination of the interchain nonlinear terms in equations (6) leads to perfect soliton switching with the period $t_0 = 100$ [figures 4(a) and 5(a)]. The inclusion of the nonlinear terms of type I leads again to perfect soliton switching but with a period $t_0 = 107$ which differs slightly from the period of the linear case [figures 4(b) and 5(b)]. The nonlinear interaction of type I has the role of an effective reduction of the coupling constant d and hence to an increase of the period. The inclusion of the nonlinear terms of type II leads also to perfect soliton switching but with a different soliton form [figures 4(c) and 5(c)]. Due to this type of nonlinear interactions the system (8) has the form of a Manakov system of coupled nonlinear equations which for $g = d/2$ is integrable and has soliton solutions. The parameters of the new solitons φ_1 and L_1

$$\varphi_1^2 = \varphi_0^2 \frac{A - d/2}{A}, \quad L_1 = L \frac{A}{A - d/2}. \quad (15)$$

can be determined using the conservation law for the number of particles of the initial excitation. This explains qualitatively our the numerical results. On figures 4(d) and 5(d) is shown the role of the interplay of both nonlinear interactions between the two chains.

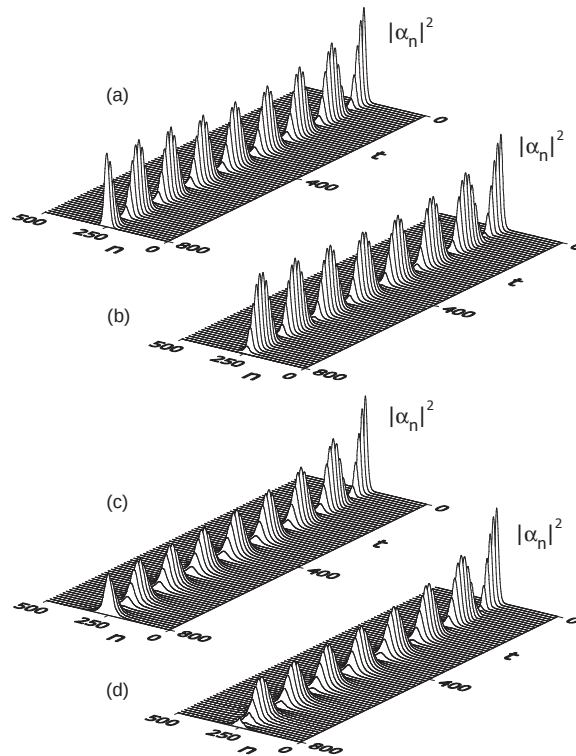


Figure 4. Soliton switching for $A = 0.025$ and $d = 0.0314$ with only linear coupling (a), linear coupling and nonlinear coupling I (b), linear coupling and nonlinear coupling II (c), linear coupling, nonlinear coupling I and II (d).

We can calculate the change of total magnetic moment in the system given by the expression

$$\Delta M = \mu S \int_{-\infty}^{\infty} (1 - \hat{S}^z(x, T)) dx, \quad (16)$$

where μ is the magnetic moment. Using (11) our calculations are performed up to the lowest approximation in ϕ_0^2

$$\Delta M = \mu S L \phi_0^2. \quad (17)$$

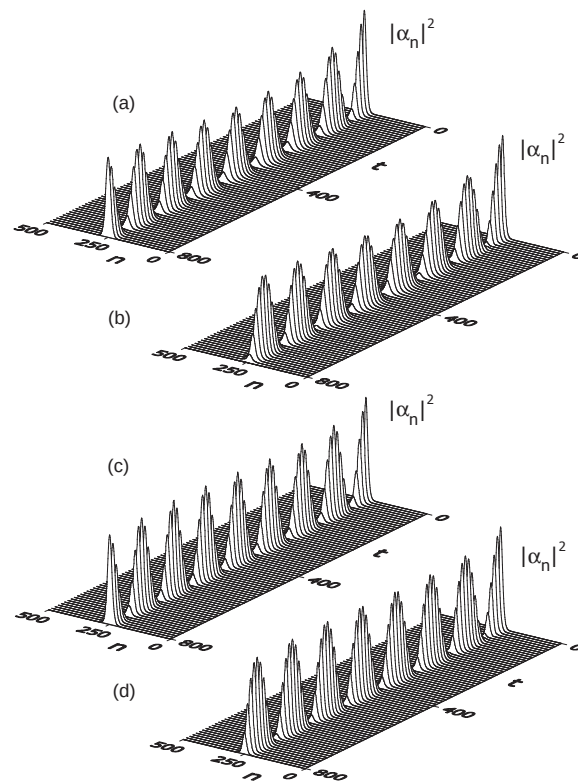


Figure 5. Soliton switching for $A = 0.025$ and $d = -0.0314$ with only linear coupling (a), linear coupling and nonlinear coupling I (b), linear coupling and nonlinear coupling II (c) and linear coupling, nonlinear coupling I and II (d).

5. Conclusion

We have investigated the propagation of a bright soliton in a system of two anisotropic ferromagnetic chains coupled with linear and nonlinear interactions. The condition for a perfect soliton switching is obtained. We have observed that in contrast to the linear interchain coupling for the nonlinear coupling not only its value but also its sign plays a crucial role for the soliton dynamics.

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